COMPUTATION OF SYMMETRIC DISCRETE COSINE TRANSFORM USING BAKHVALOV'S ALGORITHM

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ABSTRACT

A number of algorithms for recursive computation of the discrete cosine transform (DCT) have been developed recently. This paper presents a new method for computing the discrete cosine transform and its inverse using Bakhvalov's algorithm, a method developed for evaluation of a polynomial at a point. In this paper, we will focus on both the application of the algorithm to the computation of the DCT-I and its complexity. In addition, Bakhvalov's algorithm is compared with Clenshaw's algorithm for the computation of the DCT.

1. INTRODUCTION

The discrete cosine transform (DCT) is commonly used in signal processing, signal compression, image processing, and communication systems [1]. Researchers have developed efficient algorithms for computing the forward and inverse discrete transforms. These algorithms are similar to the fast Fourier transform (FFT) algorithms for computing the discrete Fourier transform (DFT). Specifically, they are more appropriate for minimizing the complexity of computations [2, 3].

Wang and Hunt have shown that for Markovian signals with correlation coefficient of less than 0.8, the symmetrical DCT (DCT-I) has better performance than the DCT-II [4]. In addition, the DCT-I has been used to compute the real discrete Fourier transform [5]. The essential computation in all these transforms is of the form

$$Y(k)=\sum_{n=0}^{N} y(n)\cos(kn\pi / N) \tag{1}$$

for $k=0,1,\ldots,N$.

This paper will focus on the recursive computation of the symmetric discrete cosine transform (DCT-I) using a computational method developed by Bakhvalov for the evaluation of an Nth order polynomial at a point [6]. This method developed for polynomial evaluation is more accurate numerically than other methods in certain cases [6]. The back–to-front computational method computes the sum of a sequence of numbers by adding small numbers first to minimize the round-off errors. Kronsjo [6] presents an upper bound for the round-off error. Bakhvalov's method requires additional adders and dividers by 2 compared with Clenshaw's methods [5, 6].

Since both Bakhvalov's and Clenshaw's algorithms are recursive, they are easily implemented on parallel computing platforms as well as VLSI and Field Programmable Gate Arrays (FPGAs).

2. Computation of the Discrete Cosine Transform

The symmetric discrete cosine transform (DCT-I) [1, 7] of N+1 data points $y(0), y(1),\ldots, y(N)$ is given by

$$Y(k)=(2/N)^{1/2}\alpha_k\sum_{n=0}^{N}\alpha_n y(n)\cos(kn\pi / N) \tag{2}$$

and its inverse is given by

$$y(n)=(2/N)^{1/2}\alpha_n\sum_{k=0}^{N}\alpha_k Y(k)\cos(nk\pi / N) \tag{3}$$

for k, n = 0, 1, ..., N,
where

$$\alpha_p = 1/\sqrt{2}$$

when $p=0$ or N and

$$\alpha_p = 1$$

when

$$p \neq 0 \text{ or } p \neq N.$$

3. BAKHVALOV'S ALGORITHM FOR EVALUATING POLYNOMIALS

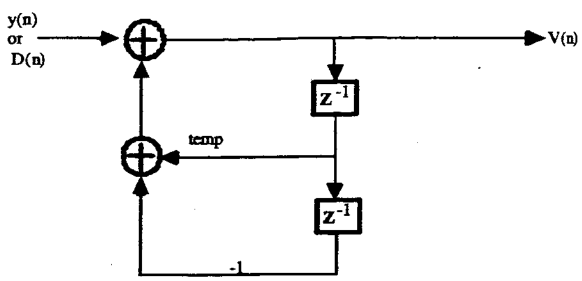

Figure 3: Recursive structure of algorithms

4. COMPUTATION OF OTHER FORMS OF DCTS

The most commonly used DCT is the DCT-II. Wang et. al. [8] showed that the DCT of N data points $y(0)$, y(1), ..., y(N-1) can be expressed as

$$Y(k) = (2/N)\ \gamma_k \sum_{n=0}^{N-1} y(N-1-n)\cos[k(n+1/2)\pi/N)] \quad (7)$$

$$y(n) = (-1)^n \sum_{k=0}^{N-1} Y(N-1-k)\sin[(k+1)(n+1/2)\pi/N] \quad (8)$$

for k,n = 0, 1, ..., N-1,
where $\gamma_k = ½$ if k=0 and 1 otherwise.

Using the relationships of equations (9) and (10), equations (7) and (8) can be put in the form of equation (2), the symmetric DCT form [2,9].

$$\sum_{k=1}^{(N+1)/2} y(n)\cos\ w(k+1/2) = \cos(w/2) \sum_{k=0}^{(N-1)/2} \tilde{y}(n)\cos(wk) \quad (9)$$

$$\sum_{k=1}^{(N+1)/2} y(n)\sin\ w(k+1/2) = \sin(w/2) \sum_{k=0}^{(N+1)/2} \tilde{y}(n)\cos(wn) \quad (10)$$

5. NUMERICAL ACCURACY COMPARISONS

The mean square errors (MSE) of Bakhvalov's and Clenshaw's methods were calculated for various types of signals using Matlab. The DCTs were computed for constants, ramps, parabolas, exponentials and random sets (uniform distributions). Trials were run on each set using 256, 512 and 1024 data points and several different ranges for the amplitudes.

We used the sorted direct implementation of equation (3) as the reference for numerical accuracy. The product terms in the direct implementation of the DCT were sorted and added from smallest to largest values to minimize round-off error when computing the partial sums.

Table 1 shows the inputs with N data points, y(0), y(1), y(2) .... Y(N-1) for which the numerical accuracy of Bakhvalov's and Clenshaw's methods were computed using Matlab, where u(n) is the unit step sequence. The entries under Bakhvalov indicate the cases where this method was more accurate than Clenshaw's. Similarly, the entries under Clenshaw indicate the cases where this method was more accurate than Bakhvalov's. Both methods were within approximately one percent of each other. It is difficult to know in advance which method will produce more accurate results.

Both Bakhvalov's and Clenshaw's methods proved to be quite accurate. One method occasionally proved to be superior for a given group of data sets, for example Clenshaw's method when used on a ramp and Bakhvalov's for unit steps of length 1024.

In addition, we tested 50 random input sequences of length 256, 512, and 1024 using uniformly distributed (0,1) values. The MSE for Clenshaw's was lower in 27 cases for length 256 and 32 cases for length 512. The MSE was lower for Bakhvalov's method in 29 cases for length 1024.

6. SUMMARY

We presented a new method for the computation of the recursive symmetric discrete cosine transform based on Bakhvalov's method for evaluating a polynomial at a point. It is a back-to-front computational method and easily implemented on parallel platforms and hardware. When compared with Clenshaw's method., the recursive part of both algorithms has the same complexity. Both Bakhvalov's and Clenshaw's methods can be used to compute other forms of discrete cosine transforms. In addition, we compared the numerical accuracy of the two methods.

7. REFERENCES

[1] K.R. Rao and P. Yip, *"Discrete Cosine Transform: Algorithms, Advantages, Applications,"* Academic Press, Inc., San Diego, 1990.

[2] L.R. Rabiner and B. Gold, *"Theory and Application of Digital Signal Processing,"* Prentice-Hall, Englewood Cliffs, NJ, 1975.